Statistical Estimation of the Received Power in a Multipath Environment

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ABSTRACT

Ray-tracing algorithms do not provide complete information on the phases of the received electromagnetic field. Generally, phases are assumed to be independent random variables, uniformly distributed in the interval $[-\pi,\pi)$ For more than six paths ($n\geq 6$) with equal power contributions, the total received power has been shown to have an exponential distribution function CDF_o. An analytical solution for the cumulative distribution function CDF_2 is derived for two paths. It shows a completely different shape than any other case with $n\geq 3$. According to the power spreading among paths, either CDF_2 or CDF_o may be used as limiting distributions to compute the confidence level or outage probability of the received power.

I. INTRODUCTION

In the past, conventional wireless systems have been planned using empirical or semi-empirical coverage prediction methods. New propagation models[1] [2], based on a ray-optical approach, make use of the uniform theory of diffraction and physical optics to deal with wave interactions like diffraction and scattering.

While the direct and diffracted paths of the received field can be analytically determined in amplitude and phase, the theories for rough surfaces scattering allow only the amplitudes of the scattered fields to be calculated in an analytic form. In the frequency range 5 to 60 GHz most surfaces appear relatively "rougher," which results in greater diffusion of the signal and less specular reflection [3]. The natural assumption of a random phase for each path stems from these considerations.

II. MODEL DESCRIPTION

In order to determine the path loss, an unmodulated carrier ω_c is transmitted. At the receiver input, the complex envelope of the multipath signal results as the sum of all path signals (1)

$$\tilde{u}_r = \sum_{k=1}^{n} U_k \, e^{j\psi_k} \, e^{-j\omega_c \tau_k} \qquad (1)$$

The set of amplitudes $(U_k)_{k=1,n}$ is related to the received field through the antenna gain. While $(U_k)_{k=1,n}$ may be computed analytically, phases $(\Psi_k)_{k=1,n}$ are supposed to be independent and identical distributed (i.i.d.) random variables (r.v.), with a uniform distribution between $[-\pi, \pi)$. One can see that the complex envelope does not depend on time when multipath signals are not frequency shifted by the Doppler effect.

To simplify notations, the phase shift produced by the excess path delay τ_k is compounded with the random phase Ψ_k. Thus, the complex envelope becomes

$$\tilde{u}_r = \sum_{k=1}^{n} U_k \, e^{j\phi_k} \qquad (2)$$

where $\Phi_k = \Psi_k + \omega_c \tau_k$.

The 'received power' random variable, $\mathbf{w}_r$, is related to the complex envelope (1) according to the equation (3)

$$\mathbf{w}_r = \tfrac{1}{2} |\tilde{\mathbf{u}}_r|^2 \qquad (3)$$

Fixing the amplitude set, the outcomes of the r.v. $\mathbf{w}_r$ depend on the phases $(\phi_k)_{k=1,n}$. In the most general case, the cumulative distribution function [4] of the r.v. $\mathbf{w}_r$ will be denoted by $CDF_n(w;U_{k=1,n})$. This depends on n parameters that are the amplitudes U_k or related powers $p_k = \frac{1}{2} U_k^2$ of each path. The expression for CDF_n may be derived analytically only for a few special cases which will be presented below. Knowing the average P_r and the standard deviation σ_w of the received power, an approximating distribution will be suggested in order to estimate the probability of the event {received power less than or equal w}, noted by $\{\mathbf{w}_r \leq w\}$.

III. PARAMETERS OF THE RECEIVED POWER

The expected value of the r.v. $\mathbf{w}_r$ represents the average received power P_r

$$P_r = \mathrm{E}\{\mathbf{w}_r\} \qquad (4)$$

where E{.} denotes statistical average.

Considering the equation (3), the variance of the r.v. $\mathbf{w}_r$ is given by equation (5)

$$\sigma_{\mathbf{w}}^{\mathbf{2}} = \mathrm{var}\left(\tfrac{1}{2} |\tilde{u}_r|^2\right) = \tfrac{1}{4} \mathrm{E}\{|\tilde{u}_r|^4\} - P_r^2 = \tfrac{1}{2} \sum_{\substack{p,q=1 \\ p\neq q}}^{n} U_p^2 U_q^2 \qquad (5)$$

When all the components of the received signal have the same power, the variance (5) reaches a maximum. The r.v. $\mathbf{w}_r$ is upper bounded and positive (6)

$$0 \le \mathbf{w}_r \le \left(\sum_{k=1}^{n} U_k \right)^2 \qquad (6)$$

IV. LIMITING CDFs

In the process of deriving an approximation for the probability of the event $\{\mathbf{w}_r \le w\}$, where w is a real constant, some useful cases will be considered.

A. Many paths with equal power

For this case, the random variable 'received power' is denoted by $\mathbf{w}$. According to [5], if amplitudes U_k are equal and $n \ge 6$, with a good approximation, the random variable $\mathbf{y}$ associated with the envelope

$$\mathbf{y} = |\tilde{\mathbf{u}}_r| \qquad (7)$$

has a Rayleigh distribution with the density function (pdf) given by

$$f_{\mathbf{y}}(y) = \frac{y}{\alpha^2} \cdot \mathrm{e}^{-\frac{y^2}{2\alpha^2}} U(y) \qquad (8)$$

where U(y) is the unit step function and α is a parameter. Considering the relationship between random variables $\mathbf{w}_r$ and $\mathbf{y}$, the pdf of the received power $f_{\mathbf{w}}$ has an exponential distribution (a Gamma distribution [2] with parameters b=0, $c=\alpha^{-1}$) (9)

$$f_{\mathbf{w}}(w) = \frac{1}{\alpha} \cdot \mathrm{e}^{-\frac{w}{\alpha}} U(w) \qquad (9)$$

The parameter α is resolved from the condition (10)

$$\alpha = \mathrm{E}\{\mathbf{w}\} = P_r \qquad (10)$$

For any positive number w, the cumulative distribution function CDF$_o(w)$ is defined as the probability of the event $\{\mathbf{w} \le w\}$ (11)

$$\mathrm{CDF}_o(w) = \mathrm{P}\{\mathbf{w} \le w\} = \int_0^w f_{\mathbf{w}}(x)dx = 1 - \mathrm{e}^{-w/P_r} \qquad (11)$$

B. Two paths

An exact analysis has been done when the received signal comes from two paths (n=2). Using the notation $\phi_\Delta = (\phi_1 - \phi_2)/2$, the received power ($\mathbf{w}_2$) is given by (12)

$$\mathbf{w}_2 = P_r + U_1 U_2 \cos(2\Phi_\Delta) \qquad (12)$$

The phase of each ray was assumed to have the expression $\phi_k = \phi_{k0} + \mathbf{x}_k$ (k=1,2), where ϕ_{k0} is a constant (due to the excess delay of the path), and $\mathbf{x}_k$ is a r.v. uniformly distributed in $[-\pi, \pi)$. First, the density function for the r.v. $2\phi_\Delta$ is derived

$$2\Phi_\Delta = \Phi_{10} - \Phi_{20} + \mathbf{x}_1 - \mathbf{x}_2 \qquad (13)$$

With the previous assumption that $\mathbf{x}_1$ and $\mathbf{x}_2$ are independent, the density of the expression $\mathbf{x} = \mathbf{x}_1 - \mathbf{x}_2$ is given by (14)

$$f_{\mathbf{x}}(x) = \begin{cases} \dfrac{1}{2\pi}\left(1 - \dfrac{|x|}{2\pi}\right) & ; |x| \le 2\pi \\ 0 & : |x| \ge 2\pi \end{cases} \qquad (14)$$

If the r.v. $\mathbf{z}$ is defined as a function of $\mathbf{x}$ (equation (15))

$$\mathbf{z} = g(\mathbf{x}) = a \cdot \sin(\mathbf{x} + \theta) \qquad (15)$$

it can be shown that the density function $f_{\mathbf{z}}(z)$ has the expression (16)

$$f_{\mathbf{z}}(z) = \frac{1}{\pi\sqrt{a^2 - z^2}} U(a - |z|) \qquad (16)$$

Using the notation $\theta = \phi_{10} - \phi_{20} - \pi/2$, equation (12) can be rewritten as (17)

$$\mathbf{w}_2 = P_r + U_1 U_2 \cdot \sin(\mathbf{x} + \theta) \qquad (17)$$

and the probability density function of the received power $\mathbf{w}_2$ (figure 1) is derived in equation (18)

$$f_2(w) = \frac{1}{\pi\sqrt{(U_1U_2)^2 - (w - P_r)^2}} U\left(U_1U_2 - |w - P_r|\right) \qquad (18)$$

This result has been verified by simulation.

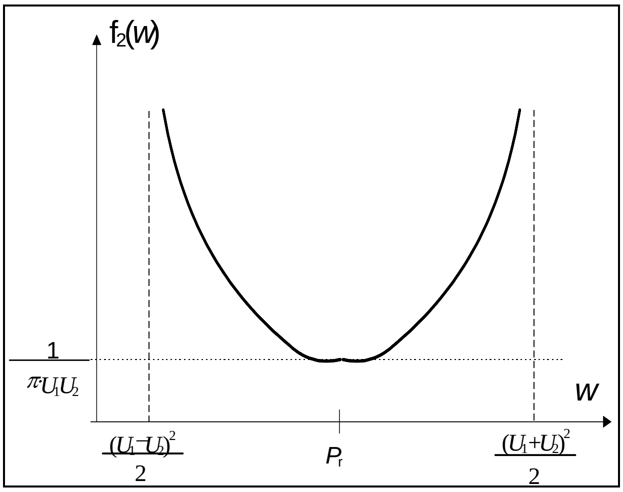

Figure 1 Density function of the random variable

The CDF$_2(w)$ is given by (19)

$$\mathrm{CDF}_2(w) = \begin{cases} 0 & : w \le P_r - U_1U_2 \\ \frac{1}{2} + \frac{1}{\pi}\arcsin\dfrac{w - P_r}{U_1U_2} & : |w - P_r| \le U_1U_2 \\ 1 & : w \ge P_r + U_1U_2 \end{cases} \qquad (19)$$

V. DISTRIBUTION FUNCTIONS

For $n \geq 3$ paths, a set of simulations have been conducted in order to study the distribution of the r.v. $\mathbf{w}_r$. For each simulation, the average power P_r is set to unity. With the substitution

$$w_{dB} = 10 \cdot \log_{10} \frac{w_r}{P_r} \qquad (20)$$

every analysis may be reduced to the unity power case considered below.

The probability of the event {$\mathbf{w}_r$ in bin_k} and the cumulative distribution function $\text{CDF}_n(\text{bin}_k)$ have been studied for different number of paths n and arrangements of path powers $p_k=(U_k)^2/2$. A number of 50 bins have been considered to draw the statistics of the r.v. $\mathbf{w}_r$. 1 dB wide bins are centered about values starting from -40 to +10dB relative to P_r.

In each graph, the functions CDF_o and CDF_2 are represented respectively with solid and dashdot lines. For the interval $-40<w_{dB}<-5$dB, CDF_o is well approximated by a straight line. It is known that the received envelope has a Rice-Nakagami distribution when one path has a larger power then the others. The weak paths are associated with a diffuse field. In the graphs below, the Rice-Nakagami distribution, depending on its parameter value, are located below CDF_o. When the power of the dominant path is very large, the corresponding CDF will be next to the 0dB vertical grid.

In figures 2 and 3 respectively the CDF_3 and the probability {$\mathbf{w}_r$ in bin_k} are represented for n=3 when two components have equal power.

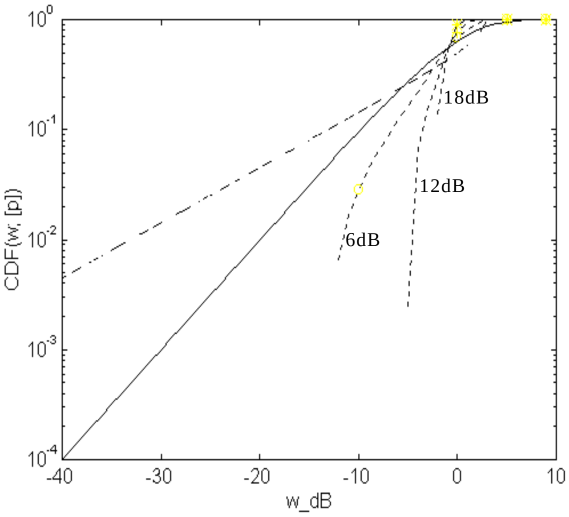

Figure 2 Distribution function for $\mathbf{w}_3$ when $p_2=p_3$ p_1 is *6, 12* or *18* dB higher than p_2+p_3.

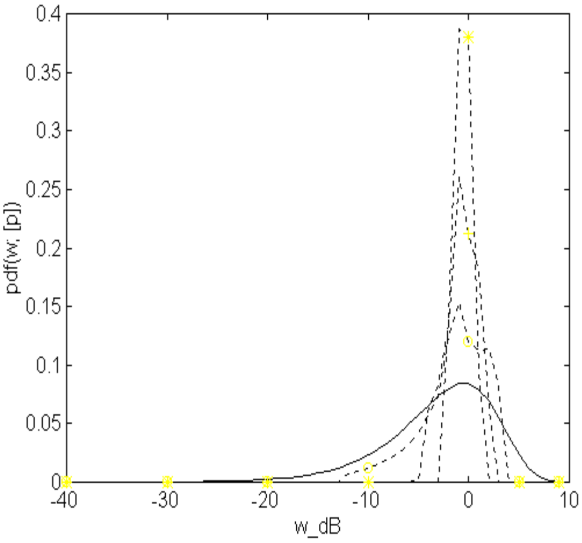

Figure 3 Density function for $\mathbf{w}_3$ when $p_2=p_3$ p_1 is *6, 12* or *18* dB higher than p_2+p_3.

For the curve that is very close to the 0 dB grid (figure 2), the dominant power is 12 dB above the combined power from the other two paths. The solid line curve in both figures represents the case 'many paths with equal power'. No further attention will be paid to the particular case of one dominant path as it is well documented in the literature.

The simulations reveal CDF_2 as a useful limiting distribution [6] for many situations where the average received power comes mainly from two paths, the power of the others being substantially lower. For example, if $p_1=p_2$ and p_1+p_2 is greater by Δp dB than the composite power of the other paths, starting from $w=\Delta p$ dB, the CDF of the received power may be approximated with CDF_2. It is important to remember that powers from weak paths have to be approximately the same.

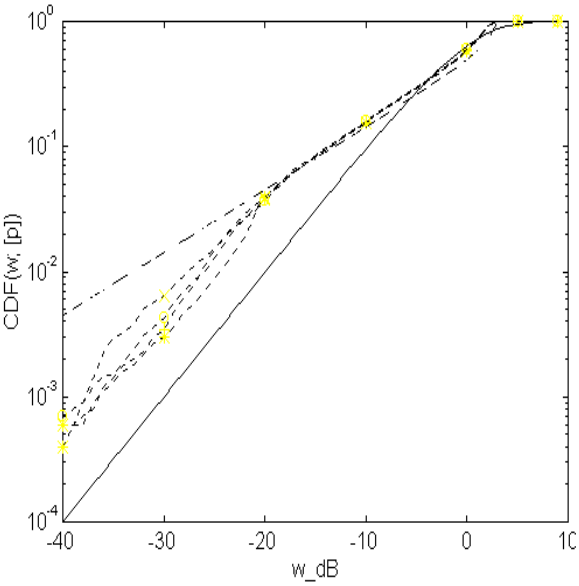

Figure 4 Distribution function for $\mathbf{w}_n$, $3 \leq n \leq 6$. $p_1=p_2$, and p_1+p_2 is 20 dB higher than the composite power of the other equal paths

In figure 4, $p_1=p_2$ and their sum is 20 dB higher than the composite power of the other equal paths. The number of paths considered for this case goes from 3 to 6 ($3 \le n \le 6$). Independent of the number of paths considered in the simulation, CDF_n is well approximated by CDF_2 for $-20dB < w_{dB}$ (figure 5).

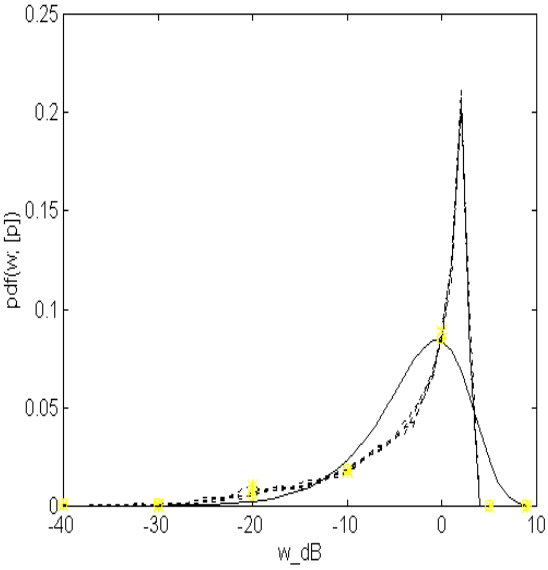

Figure 5 Density function for $\mathbf{w}_n$, $3 \le n \le 6$. $p_1=p_2$, and p_1+p_2 is 20 dB higher than the composite power of the other equal paths

A number of simulations have been run to quantify the CDF dependence on Δp. Six paths are considered in figure 6. The power coming from two equal paths is 10, 15 and 20 dB higher than the composite power of the others.

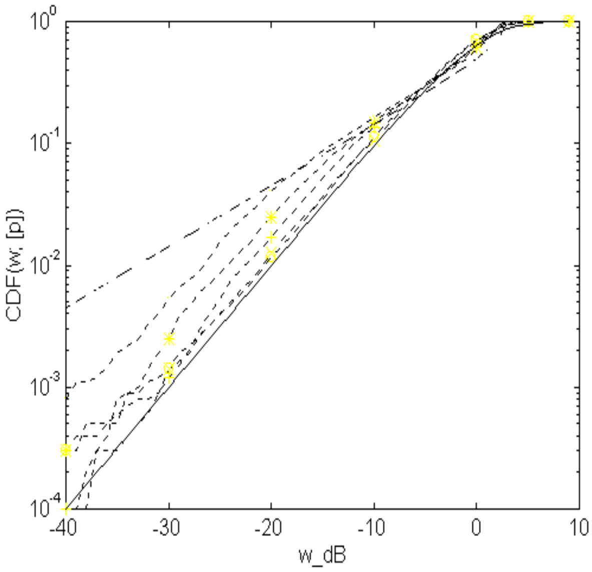

Figure 6 Distribution function for $\mathbf{w}_6$.
p_1+p_2 is 10, 15, and 20 dB higher than $p_3+..+p_6$

A distinctive case appears when path powers are equally spaced by δ dB. For $\delta<3dB$, the resultant CDF is very close to CDF_o for $w<0dB$, independently of the number of paths (figure 7).

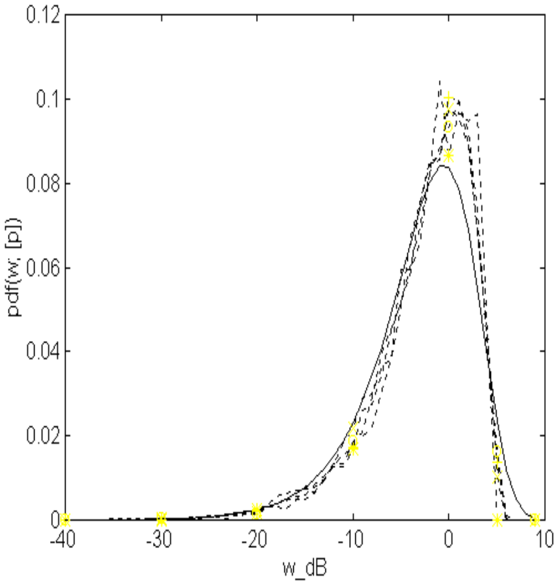

Figure 7 Density function for $\mathbf{w}_n$, $3 \le n \le 6$. path powers are equally spaced by δ=3dB

By contrast, for $\delta \ge 6dB$, the resultant CDF will be located in the Rice-Nakagami region on the graph. In figure 8, the CDF_4 function located below the CDF_o corresponds to four paths separated by 6dB.

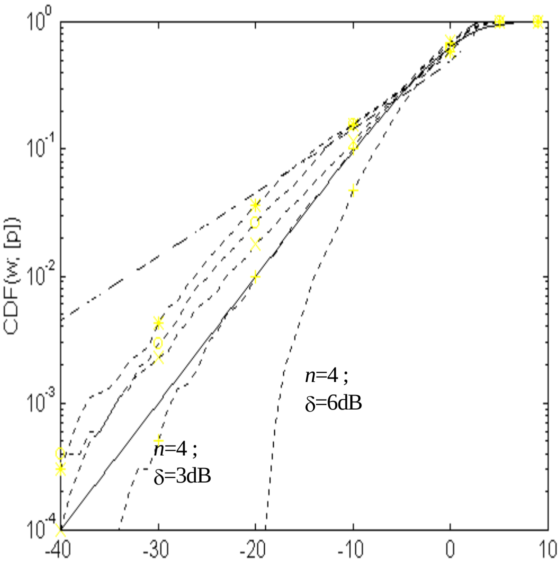

Figure 8 Density function for $\mathbf{w}_4$

VI. CONCLUSIONS

When many paths are combined, according to the number of paths and their contribution to the total average received power, different approaches are recommended to compute the probability of the event $[\mathbf{w}_r \le w]$. If the cumulative power of two equal strength paths is ΔdB greater then the power received from the others, CDF_2 is recommended for computation of the probability for $w_{dB} > -\Delta dB$. When the path powers are equally spaced by δdB, depending on δ, the resultant distribution will be close to the Gamma-exponential distribution or will be approximated by a Rice-Nakagami distribution with a proper parameter.

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